M A T H E M A T I C S

(DAILY PRACTICE PAPER)   **[CLASS XII]**

DEFINITE INTEGRAL

✤ PRACITCE QUESTION

Very Short (Objective Type) / Short Answer Type

1. If a is such that $\int_0^a x\, dx \leq a + 4$, then
   (a) $0 \leq a \leq 4$ (b) $-2 \leq a \leq 0$
   (c) $a \leq -2$ or $a \leq 4$ (d) $-2 \leq a \leq 4$

2. If $\int_1^4 f(x)\, dx = 4$ and $\int_2^4 (3 - f(x))\, dx = 7$, then the value of $\int_1^2 f(x)dx$ is
   ____.

   Evaluate the following integrals in Exercises 3 to 7:
3. $\int_0^1 (2x)/(1 + x^2)\, dx$

4. $\int_2^3 (1/x)\, dx$ [Delhi 2012]

5. $\int_0^3 (dx)/(9 + x^2)$ [Delhi 2014]

6. $\int_0^{\pi/2} \cos 3x \cos 2x\, dx$

7. $\int_0^1 (e^x)/(1 + e^{2x})\, dx$

8. If $\int_0^1 (3x^2 + 2x + k)dx = 0$, find the value of k.

Long Answer I / Long Answer II Type

Evaluate the following integrals in Exercises 9 to 22:

9. $\int_0^{\pi/4} (\sin x + \cos x)/(9 + 16 \sin 2x)\, dx$ [NCERT; DoE]

10. $\int_1^2 (1)/(x(1 + \log x)^2)\, dx$ [Foreign 2014, 2011]

11. $\int_0^a \sqrt{(a^2 - x^2)}\, dx$

12. $\int_0^1 x \sin^{-1} x \, dx$

13. $\int_0^1 x/(1 + x^4) \, dx$

14. $\int_0^{\pi/4} 1/(1 + \tan x) \, dx$

15. $\int_0^1 \tan^{-1}(2x/(1 - x^2)) \, dx$

16. $\int_0^1 x \log(1 + 2x) \, dx$

17. $\int_0^{\pi/2} e^x((1 - \sin x)/(1 - \cos x)) \, dx$

18. $\int_0^{\pi/4} \sin x \cos x/(\cos^2 x + \sin^4 x) \, dx$

19. $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x) \, dx$

20. $\int_0^1 dx/(e^x + e^{-x})$

21. $\int_0^{\pi/4} dx/(\cos^3 x \sqrt{2} \sin 2x)$

22. $\int_0^{\pi/2} dx/(a^2 \cos^2 x + b^2 \sin^2 x)$

✳ VERY SHORT (OBJECTIVE TYPE) / SHORT ANSWER TYPE QUESTION

1. $\int_{-1}^1 (x^3/x^3 + 1) \, dx$ is equal to _____.

2. $\int_a^{2a} f(x) dx = 2f(x) dx$ if $f(2a-x) = f(x)$. State true or false.

3. $\int_0^{\pi/2} (\sqrt[3]{\sin x} / \sqrt[3]{\sin x} + \sqrt[3]{\cos x}) \, dx = a * \pi/2$, then value of a is __.

4. Evaluate $\int_{-\pi/2}^{\pi/2} \sin^5 x \, dx$

5. Evaluate $\int_{-1/2}^{1/2} \cos x \log (1+x/1-x) dx$

6. Evaluate $\int_{-1}^1 x^3 /(1+x^2) \, dx$ [HOTS]

7. Evaluate $\int_{-\pi/4}^{\pi/4} x^3 \sin^4 x \, dx$

8. Evaluate $\int_{-1}^{1} |x|\, dx$

9. Evaluate: $\int_0^{2\pi} \cos^5 x\, dx$

10. Evaluate $\int_0^{\pi/2} \log(\tan x)\, dx$

11. Evaluate $\int_0^1 [2x]\, dx$

12. Evaluate $\int_1^4 f(x)\, dx$, where
    $f(x) = \{2x+8, 1 \le x \le 2; 6x, 2 \le x \le 4\}$

13. Evaluate $\int_{-1}^1 \log\left|(2+3x) / (2-3x)\right|\, dx$

Long Answer I / Long Answer II Type

14. Evaluate $\int_0^{\pi} \left(e^{(\cos x)} / (e^{(\cos x)} + e^{-\cos x})\right) dx$

15. Evaluate $\int_0^1 \cot^{-1}(1-x+x^2)\, dx$

16. Evaluate $\int_{-1}^2 |x^3 - x|\, dx$ [NCERT; Delhi, 2016; AI 2012]

17. Evaluate $\int_0^{\pi} (x \tan x) / (\sec x \csc x)\, dx$

18. Evaluate $\int_0^{\pi/2} \log(\sin x)\, dx$

19. Evaluate $\int_0^{\pi} \left(x / (a^2 \cos^2 x + b^2 \sin^2 x)\right) dx$

20. Evaluate $\int_0^1 \log(1+x) / (1+x^2)\, dx$

21. Evaluate $\int_0^{\pi} x \tan x / (\sec x + \tan x)\, dx$.

22. Evaluate $\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x)\, dx$ [NCERT: DoE]

23. Evaluate $\int_0^1 \log((1-x)/x)\, dx$ **OR** $\int_0^1 \log(1/x - 1)\, dx$

24. Evaluate $\int_0^{2x} 1 / (1+e^{\sin x})\, dx$

25. Prove that $\int_0^{\pi/4} \log(1+\tan\theta)\, d\theta = (\pi/8)\log 2$

26. Evaluate $\int_0^2 |x^2 + 2x - 3|\, dx$

27. Evaluate $\int_0^1 \log x / \sqrt{(1-x^2)}\, dx$

28. Evaluate: $\int_0^{-3/2} |x \sin \pi x|\, dx$

29. Evaluate $\int_{\pi/6}^{\pi/3} dx / (1 + \sqrt{\cot x})$

30. Evaluate $\int_1^3 |x^3 - x|\, dx$

31. Evaluate $\int_0^{\pi/2} x \sin x \cos x / (\sin^4 x + \cos^4 x)\, dx$

36. Prove that $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$ and hence, prove that
$\int_0^{\pi/2} (\sin x / (\sin x + \cos x))\, dx = \pi/4$

�֍ PRACTICE QUESTION

✤ Very Short (Objective Type) / Short Answer Type

1. If $\int_0^1 (1 / (1 + 4x^2))\, dx = \pi/8$, then a = ___.
2. $\int_{-1}^0 |(1 - x)|\, dx$ is equal to ___.
3. $\int_{-\pi/2}^{\pi/2} (\sqrt{\cos x} - \cos^3 x)\, dx$ is equal to 0. State true or false.
4. The value of $\int_0^{\pi} |\cos x|\, dx$ is 2. State true or false.
5. The value of $\int_{-\pi}^{\pi} \sin^3 x \cos^2 x\, dx$ is ___.

Evaluate the following integrals in Exercises 6 to 15:

6. $\int_{-\pi}^{\pi} (\sin -93x + x^{295})\, dx$
7. $\int_{-\pi/2}^{\pi/2} \log |(2 - \sin x) / (2 + \sin x)|\, dx$
8. $\int_{-\pi/4}^{\pi/4} |\sin x|\, dx$ [DoE]
9. $\int_0^{\pi/2} ((\sin x - \cos x) / (1 + \sin x \cos x))\, dx$
10. $\int_1^e \log x \cdot dx$
11. $\int_0^1 x(1 - x)^{89}\, dx$
12. $\int_0^a (f(x) / (f(x) + f(a - x)))\, dx$ [Dehradun 2019]

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13. $\int_0^1 x (1 - x)^n \, dx$

14. $\int_1^3 \sqrt{(4 - x)} / (\sqrt{x} + \sqrt{(4 - x)}) \, dx$ [DoE]

15. $\int_0^\pi |\cos x| \, dx$ [DoE]

✤ <u>Long Answer 1 / Long Answer II Type</u>

<u>Evaluate the following integrals in Exercises 16 to 28:</u>

16. $\int_0^{3/2} |x \cos \pi x| \, dx$ [AI 2016; Patna 2015]

17. $\int_1^4 (|x - 1| + |x - 2| + |x - 4|) \, dx$

18. $\int_{-5}^5 |x + 2| \, dx$

19. $\int_{-1}^{3/2} |x \sin \pi x| \, dx$ [NCERT; DoE]

20. $\int_{-\pi}^\pi (\cos ax - \sin bx)^2 \, dx$ [Delhi 2015]

21. $\int_0^{\pi/2} x \cot x \, dx$ [HOTS]

22. $\int_{\pi/6}^{\pi/3} (1 / (1 + \sqrt{\tan x})) \, dx$ [DoE; HOTS]

23. $\int_0^8 |x - 5| \, dx$

24. $\int_2^8 3\sqrt{x+1} / 3\sqrt{x+1} + 3\sqrt{11-x} \, dx$

25. $\int_{-\pi/2}^{\pi/2} [\sin|x| - \cos|x|] \, dx$ [DoE]

26. $\int_0^{\pi/2} \sqrt{\cot x} / \sqrt{\cot x} + \tan x \, dx$ [NCERT]

27. $\int_0^\pi x \sin x / 1 + \cos^2 x \, dx$

        [NCERT; DoE; Delhi 2017(C), AI 2013, 12]

28. $\int_0^\pi x / 1 + \sin x \, dx$ [NCERT; NCERT Exemplar; DoE]

❊ INTEGRATED EXERCISE

✤ <u>Very Short (Objective Type) / Short Answer Type</u>

1. $\int (1/x^2 (x^4+1)^{3/4}) \, dx$ is equal to
   (a) $-(1 + 1/x^4)^{1/4} + C$
   (b) $(x^4+1)^{3/4} + C$
   (c) $(1 - 1/x^4)^{1/4} + C$
   (d) $-(1 + 1/x^4)^{3/4} + C$

2. $\int xe^x / (1+x)^2 \, dx$ is equal to
   (a) $e^x / x + 1 + C$
   (b) $e^x (x+1) + C$
   (c) $-e^x / (x+1)^2 + C$
   (d) $e^x / 1 + x^2 + C$

3. If $\int 2^x/\sqrt{(1-4^x)}\, dx = p \sin^{-1}(2^x) + C$, then 'p' is equal to
   (a) $\log_e 2$
   (b) $1/2 \log_e 2$
   (c) $1/2$
   (d) $1/\log_e 2$

4. The value of integral $\int (\pi/4, 0)\, (\sin x + \cos x)/(9 + 16 \sin 2x)\, dx$ is
   (a) $\log 2$
   (b) $1/20 \log 2$
   (c) $1/20 \log 3$
   (d) $\log 5$

5. The value of integral $\int (1/2, 1)\, \cos x . \log((1+x)/(1-x))\, dx$ is
   (a) 0   (b) ½   (c) 3/2   (d) none of these

6. If [x] denotes the greatest integer less than or equal to x, then the value of the integral $\int (2, 0)\, x^2 [x] dx$ is _____ .

7. $\int^3_1 \sqrt{x} / [\sqrt{(6-x)} + \sqrt{x}]\, dx$ is equal to 2. State true or false.

Evaluate the following in Exercises 8 to 19:

8. $\int_0^{\pi/2} \sin 2x\, dx$ [Foreign 2014]

9. $\int e^x / (1 + e^{2x})\, dx$

10. $\int x / \sqrt{x} + 2\, dx$

11. $\int 1 / (\cos^2 x (1 - \tan x)^2)\, dx$

12. $\int \tan^{-1}(\cot x)\, dx$ [HOTS]

13. $\int \cos 3x \cos x\, dx$

14. $\int \sin 3x \sin 2x\, dx$

15. $\int_2^4 x / x^2 + 1\, dx$ [AI 2014]

16. $\int e^2 / x \log x \, dx$ [AI 2014]

17. $\int_0^\pi (\sin^2 x/2 - \cos^2 x/2) \, dx$ [NCERT]

18. $\int_{-\pi}^\pi x^{10} \sin^7 x \, dx$

19. $\int \{\sin(\log x) + \cos(\log x)\} \, dx$

20. Given $\int e^x(\tan x + 1) \sec x \, dx = e^x f(x) + C$. Write the value of f(x).
    [AI 2012]

21. If $\int ((x-1)/x^2) e^x \, dx = f(x)e^x + C$, then write the value of f(x).
    [Foreign 2014]

22. If $\int_0^a 3 \, x^2 dx = 8$ write the value of a.   [Foreign 2014]

✤   <u>Long Answer I / Long Answer II Type</u>
   <u>Evaluate the following in Exercises 23 to 83:</u>

23. $\int dx / (\sqrt{2x+1} + \sqrt{2x+2}) \, dx$

24. $\int 1 / (\sin(x-p)\cos(x-q)) \, dx$

25. $\int \sin x / \sin(x-a) \, dx$

26. $\int (x+3) \sqrt{3-4x-x^2} \, dx$ [Delhi 2015]

27. $\int dx / (\sin x + \sin 2x)$ [Delhi 2015]

28. $\int (1-\cos x) / (\cos x (1+\cos x)) \, dx$ [Chennai 2015]

29. $\int x(\log x)^2 \, dx$ [NCERT]

30. $\int (\sin^8 x - \cos^8 x) / (1-2 \sin^2 x \cos^2 x) \, dx$ [NCERT]

31. $\int (\sin^6 x + \cos^6 x) / (\sin^2 x \cos^2 x) \, dx$
    [NCERT Exemplar; DoE; Delhi 2014]

32. $\int \sin^3 x \cos^3 x \, dx$

33. $\int dx / x(x^5 + 2)$

34. $\int 1 / x(x^4 + 1) \, dx$ [NCERT; HOTS]

35. $\int_0^{\pi/2} (\sin x + \cos x) / (3 + \sin 2x) \, dx$ [Ajmer 2015]

36. $\int e^x (\sin^{-1} x + 1 / \sqrt{1-x^2}) \, dx$

37. $\int \log (1+x^2) \, dx$

38. $\int x \tan^{-1} x \, dx$ [NCERT]

39. $\int 1 / (\cos(x+a) \cos(x+b)) \, dx$

40. $\int (x^2 + 1) / ((x^2 + 4)(x^2 + 25)) \, dx$ [Delhi 2013]

41. $\int e^x(\sec x + \sec x \tan x) \, dx$ [NCERT; HOTS]

42. $\int (x^2-3x+1) / \sqrt{1-x^2} \, dx$ [Delhi 2015]

43. $\int 2x / ((1+x^2)(3+x^2)) \, dx$ [NCERT]

44. $\int 1 / (1 + \cot x) \, dx$ [NCERT]

45. $\int 1 / x^3(x^4+1)^3/^4 \, dx$

46. $\int \tan^5 x \sec^4 x \, dx$ [NCERT Exemplar]

47. $\int \cos 2x / (\cos x + \sin x)^2 \, dx$ [NCERT]

48. $\int_0^{\pi/2} \cos^2 x / (1 + 3 \sin^2 x) \, dx$ [Ajmer 2015]

49. $\int -\pi/2 \cos x / (1 + e^x) \, dx$

50. $\int \sqrt{(a-x)/(a+x)} \, dx$

51. $\int \sqrt{\cot x} \, dx$

52. $\int e^x (\log x + 1/x)\, dx$

53. $\int [\log(\log x) + 1/(\log x)^2]\, dx$ [NCERT; Bhubaneswar]

54. $\int \sin^{-1}(2x/(1 + x^2))\, dx$

[NCERT]

55. $\int 1/(\cos^4 x + \sin^4 x)\, dx$

[AIMS]

56. $\int_0^{\pi/2} \sin 2\theta/(\sin^4 \theta + \cos^4 \theta)\, d\theta$

[IIT]

57. $\int_0^{\pi/2} \sqrt{\sin \varphi} \, \cos^5 \varphi \, d\varphi.$

[NCERT; HOTS]

58. $\int 1/(\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x)\, dx$

[AIIMS]

59. $\int x \cos^{-1} x / \sqrt{1 - x^2}\, dx$

[MET]

60. $\int \log x/(x + 1)^2\, dx$

[AIMS]

61. $\int (3x - 2)\sqrt{(x^2 + x + 1)}\, dx$

[Foreign 2015]

62. $\int e^{2x} \sin(3x + 1)\, dx$

[Foreign 2015]

63. $\int (\sqrt{\cot x} + \sqrt{\tan x})\, dx$

[NCERT: Patna 2015]

64. $\int (x^3 - 1)/(x^3 + x)\, dx$

[Patna 2015]

65. $\int \cos^{-1} (\sin x)\, dx$

[Delhi 2014]

66. $\int_0^{\pi/2} e^x (\sin x - \cos x)\, dx$

[Delhi 2014]

67. $\int_0^{\pi/2} e^x / (x \log x)\, dx$

[AI 2014]

68. $\int_0^+ 2^{\sin x}/(2^{\sin x} + 2^{\cos x})\, dx$

[Patna 2015]

69. $\int_0^1 x\, e^{(x^2)}\, dx$

[NCERT; Foreign 2014]

70. $\int_{-1}^{2} f(x)\, dx$, where $f(x) = |x + 1| + |x| + |x - 1|$

[NCERT Example]

71. $\int \sqrt{(1+x^2)}\, dx$ [Panchkula 2015]

72. $\int (x+3)e^x / (x+5)^2\, dx$ [NCERT Exemplar]

73. $\int_{0}^{\pi} x \sin x \cos^2 x\, dx$ [NCERT]

74. $\int_{0}^{\pi/2} \sin^{3/2} x / \sin^{3/2} x + \cos^{3/2} x\, dx$ [NCERT]

75. $\int_{0}^{1} [x]\, dx$ [NCERT: HOTS]

76. $\int_{0}^{1} x(1-x)^n\, dx$

77. $\int_{0}^{\pi/4} \sec x / 1 + 2 \sin^2 x\, dx$ [Guwahati 2015]

78. $\int_{0}^{\pi/4} (1 - \sin 2x) / 1 - \cos 2x\, dx$ [Guwahati 2015]

79. $\int_{0}^{\pi/2} 5 \sin x + 3 \cos x / \sin x + \cos x\, dx$ [Bhubneshwar 2015]

80. $\int x / 1 + \tan x\, dx$ [Bhubneshwar 2015]

81. $\int \sin x - x \cos x / x(x + \sin x)\, dx$ [Ajmer 2015]

82. $\int_{0}^{\pi} \sin^2 x / \sin x + \cos x\, dx$

[NCERT Exemplar; Al 2016; Panchkula 2015]

83. $\int \csc^3 x\, dx$ [HOTS]

84. Evaluate $\int_{1}^{3} (2x^2 + x + 9)\, dx$ as the limit of a sum. [HOTS]

85. Evaluate $\int_{0}^{2} (x^2 + e^{2x} + 1)\, dx$ as the limit of a sum.

[Chennai 2015]

86. $\int_{-1}^{2} (e^{3}x + 7x - 5)\, dx$ as a limit of sums. [Panchkula 2015]

87. Evaluate $\int_{1}^{3} (e^{-3x} + x^2 + 1)\, dx$ as a limit of sums. [Delhi 2015]

88. Prove that $\int_{0}^{1} \sqrt{1 - x} / 1 + x\, dx = \pi/2 - 1$.

89. If $\int_0^a 1/4 + x^2 \, dx = \pi/8$, find the value of a.   [AI 2014]

90. Prove that $\int_0^\pi x \sin x / 1 + \sin x \, dx = \pi(\pi - 2)$.

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